

# Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

# 914054992

#### **FURTHER MATHEMATICS**

9231/21

Paper 2 Further Pure Mathematics 2

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

#### **INSTRUCTIONS**

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined pages at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

#### **INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 20 pages. Any blank pages are indicated.

	s of the equation and $0 < \theta < 2\pi$ .				
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Show that $\frac{\mathrm{d}y}{}$ –	$t + \sqrt{1 - t^2} \cos^{-1} t.$	
Show that $\frac{dx}{dx} = -\frac{1}{2}$	$+$ $\vee$ 1 – $t$ $\cos t$ .	
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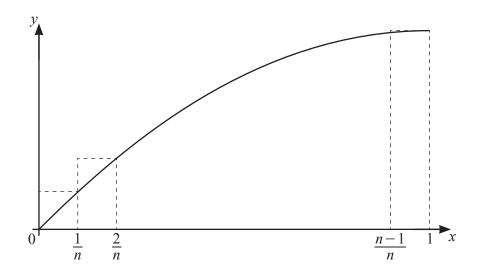
	$\operatorname{nd} \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} \text{ in terms of } t.$	
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- 4 It is given that, for  $n \ge 0$ ,  $I_n = \int_0^{\ln 3} \operatorname{sech}^n x \, dx$ .
  - (a) Show that, for  $n \ge 2$ ,

$(n-1)I_n = \left(\frac{3}{5}\right)^{n-2} \left(\frac{4}{5}\right) + (n-2)I_{n-2}.$	[5]
[You may use the result that $\frac{d}{dx}(\operatorname{sech} x) = -\tanh x \operatorname{sech} x$ .]	

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5



The diagram shows the curve with equation  $y = 2x - x^2$  for  $0 \le x \le 1$ , together with a set of n rectangles of width  $\frac{1}{n}$ .

(a)	By considering the sum of the areas of these rectangles, show that	$\int_0^1 (2x - x^2) \mathrm{d}x < U_n, \text{ where }$	
	$U_n = \left(1 + \frac{1}{n}\right)\left(\frac{2}{3} - \frac{1}{6n}\right).$	[.	5

	$\operatorname{nd} L_n \text{ for } \int_0^1 (2x - x^2) dx.$
Show that $\lim_{n \to \infty} (U_n - L_n) = 0$ .	
, , , ,	

6	(a)	Show that $\left(\cosh x + \sinh x\right)^{\frac{1}{2}} = e^{\frac{1}{2}x}$ .	[2]
	(b)	Find the particular solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5\left(\cosh x + \sinh x\right)^{\frac{1}{2}},$	
		given that, when $x = 0$ , $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$ .	[10]

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$x\frac{dy}{dx} - y = x^2 \sinh^{-1}x,$ given that $y = 1$ when $x = 1$ . Give your answer in the form $y = f(x)$ . [10]		$\int \frac{x}{\sqrt{1+x^2}}  \mathrm{d}x.$	[2]
$x\frac{dy}{dx} - y = x^2 \sinh^{-1}x,$ given that $y = 1$ when $x = 1$ . Give your answer in the form $y = f(x)$ . [10]			
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	(b)		
	(b)	$x\frac{\mathrm{d}y}{\mathrm{d}x} - y = x^2 \sinh^{-1}x,$	[10]
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(a)	Find the set of values of a for which the system of equations									
	6x + ay = 3,									
	2x-y = 1,									
	x + 5y + 4z = 2									
	has a unique solution.	[2								
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(b)	Show that the system of equations in part (a) is consistent for all values of $a$ .	[3								
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The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 6 & 0 & 0 \\ 2 & -1 & 0 \\ 1 & 5 & 4 \end{pmatrix}.$$

Fin	ıd a matı	rix <b>P</b> ai	nd a d	ıagona	I matr	ıx <b>D</b> s	such th	nat (1	4 <b>A</b> + 2	241) =	= PDP	•			[7
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l)	Jse the characteristic equation of <b>A</b> to show that	
	$\left(14\mathbf{A} + 24\mathbf{I}\right)^2 = \mathbf{A}^4 \left(\mathbf{A} + b\mathbf{I}\right)^2,$	
	where b is an integer to be determined. [4	]
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# Additional page

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